

# Mark Scheme 4725

## June 2007

1	<p><i>EITHER</i>  <math>a = 2</math></p> <p><math>b = 2\sqrt{3}</math>,  <i>OR</i></p> <p><math>a = 2 \quad b = 2\sqrt{3}</math></p>	<p>M1            A1            M1            A1            M1            M1            A1 A1</p>	<p>4</p> <p>4</p>	<p>Use trig to find an expression for <math>a</math> (or <math>b</math>)            Obtain correct answer            Attempt to find other value            Obtain correct answer a.e.f.            (Allow 3.46 )            State 2 equations for <math>a</math> and <math>b</math></p> <p>Attempt to solve these equations            Obtain correct answers a.e.f.            SR <math>\pm</math> scores A1 only</p>
2	<p><math>(1^3 = ) \frac{1}{4} \times 1^2 \times 2^2</math></p> <p><math>\frac{1}{4} n^2(n+1)^2 + (n+1)^3</math></p> <p><math>\frac{1}{4} (n+1)^2(n+2)^2</math></p>	<p>B1            M1            M1(indep)            A1            A1</p>	<p>5</p> <p>5</p>	<p>Show result true for <math>n = 1</math></p> <p>Add next term to given sum formula            Attempt to factorise and simplify            Correct expression obtained convincingly</p> <p>Specific statement of induction conclusion</p>
3	<p><math>3\sum r^2 - 3\sum r + \sum 1</math></p> <p><math>3\sum r^2 = \frac{1}{2} n(n+1)(2n+1)</math></p> <p><math>3\sum r = \frac{3}{2} n(n+1)</math></p> <p><math>\sum 1 = n</math>  <math>n^3</math></p>	<p>M1            A1            A1            A1            M1            A1</p>	<p>6</p> <p>6</p>	<p>Consider the sum of three separate terms</p> <p>Correct formula stated</p> <p>Correct formula stated</p> <p>Correct term seen            Attempt to simplify            Obtain given answer correctly</p>
4	<p>(i) <math>\frac{1}{2} \begin{pmatrix} 5 &amp; -1 \\ -3 &amp; 1 \end{pmatrix}</math></p> <p>(ii)</p> <p><math>\frac{1}{2} \begin{pmatrix} 2 &amp; 0 \\ 23 &amp; -5 \end{pmatrix}</math></p>	<p>B1            B1            M1            M1(indep)            A1ft            A1ft</p>	<p>2</p> <p>4</p> <p>6</p>	<p>Transpose leading diagonal and negate other diagonal or solve sim. eqns. to get 1<sup>st</sup> column            Divide by the determinant or solve 2<sup>nd</sup> pair to get 2<sup>nd</sup> column</p> <p>Attempt to use <math>B^{-1}A^{-1}</math> or find B            Attempt at matrix multiplication            One element correct, a.e.f.            All elements correct, a.e.f.            NB ft consistent with their (i)</p>

5	<p>(i) <math>\frac{1}{r(r+1)}</math></p> <p>(ii) <math>1 - \frac{1}{n+1}</math></p> <p>(iii) <math>S_{\infty} = 1</math> <math>\frac{1}{n+1}</math></p>	<p>B1</p> <p>M1 M1 A1</p> <p>B1 ft M1 A1 c.a.o.</p>	<p>1 Show correct process to obtain given result</p> <p>3 Express terms as differences using (i) Show that terms cancel Obtain correct answer, must be <math>n</math> not any other letter</p> <p>3 State correct value of sum to infinity Ft their (ii) Use sum to infinity – their (ii)</p> <p>7 Obtain correct answer a.e.f.</p>
6	<p>(i) (a) <math>\alpha + \beta + \gamma = 3, \alpha\beta + \beta\gamma + \gamma\alpha = 2</math></p> <p>(b)</p> <p><math>\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \beta\gamma + \gamma\alpha)</math> <math>= 9 - 4 = 5</math></p> <p><math>\frac{3}{u^3} - \frac{9}{u^2} + \frac{6}{u} + 2 = 0</math></p> <p>(ii) (a) <math>2u^3 + 6u^2 - 9u + 3 = 0</math></p> <p><math>\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = -3</math></p> <p>(b)</p>	<p>B1 B1</p> <p>M1</p> <p>A1 ft</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1 ft</p>	<p>2 State correct values</p> <p>2 State or imply the result and use their values</p> <p>2 Obtain correct answer</p> <p>2 Use given substitution to obtain an equation</p> <p>2 Obtain correct answer</p> <p>8 Required expression is related to new cubic stated or implied -(their “b” / their “a”)</p>

7	(i) $a(a - 12) + 32$ (ii) $\det \mathbf{M} = 12$ non-singular (iii) <i>EITHER</i>  <i>OR</i>	M1 M1 A1  M1 A1ft B1  M1  A1  M1 A1 A1	3  2  3	Show correct expansion process Show evaluation of a 2 x 2 determinant Obtain correct answer a.e.f.  Substitute $a = 2$ in their determinant  Obtain correct answer and state a consistent conclusion  $\det M = 0$ so non-unique solutions  Attempt to solve and obtain 2 inconsistent equations Deduce that there are no solutions  Substitute $a = 4$ and attempt to solve Obtain 2 correct inconsistent equations Deduce no solutions
8	(i) Circle, centre (3, 0), $y$ -axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1 <sup>st</sup> quadrant only (ii) Inside circle, below line, above $x$ -axis	B1B1 B1 B1 B1 B1 B2ft	6 2 <b>8</b>	Sketch showing correct features N.B. treat 2 diagrams as MR  Sketch showing correct region SR: B1ft for any 2 correct features

9	<p>(i) <math>\begin{pmatrix} \sqrt{2} &amp; 0 \\ 0 &amp; \sqrt{2} \end{pmatrix}</math></p> <p>(ii) Rotation (centre <math>O</math>), <math>45^\circ</math>, clockwise</p> <p>(iii)</p> <p>(iv) <math>\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ -1 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix}</math></p> <p>(v) <math>\det C = 2</math> area of square has been doubled</p>	<p>B1</p> <p>B1B1B1</p> <p>B1</p> <p>M1 A1</p> <p>B1</p> <p>B1</p>	<p>1</p> <p>3</p> <p>1</p> <p>2</p> <p></p> <p>2</p> <p><b>9</b></p>	<p>Correct matrix</p> <p>Sensible alternatives OK, must be a single transformation</p> <p>Matrix multiplication or combination of transformations</p> <p>For at least two correct images For correct diagram</p> <p>State correct value</p> <p>State correct relation a.e.f.</p>
10	<p>(i)</p> <p><math>x^2 - y^2 = 16</math> and <math>xy = 15</math></p> <p><math>\pm(5 + 3i)</math></p> <p>(ii)</p> <p><math>z = 1 \pm \sqrt{16 + 30i}</math></p> <p><math>6 + 3i, -4 - 3i</math></p>	<p>M1</p> <p>A1A1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>M1*</p> <p>A1 *M1dep A1 A1ft</p>	<p></p> <p></p> <p></p> <p></p> <p>6</p> <p></p> <p>5</p> <p><b>11</b></p>	<p>Attempt to equate real and imaginary parts of <math>(x + iy)^2</math> and <math>16 + 30i</math></p> <p>Obtain each result</p> <p>Eliminate to obtain a quadratic in <math>x^2</math> or <math>y^2</math></p> <p>Solve to obtain <math>x = (\pm) 5</math> or <math>y = (\pm) 3</math></p> <p>Obtain correct answers as complex numbers</p> <p>Use quadratic formula or complete the square</p> <p>Simplify to this stage Use answers from (i) Obtain correct answers</p>

